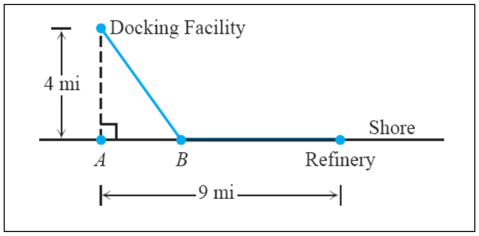
AP Calculus AB — Homework 4.1c — Name:

Dr. Paul L. Bailey — Wednesday, December 12, 2019

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. In particular, you *must* write any function the problem refers to.

Problem 1 (Thomas §4.1 # 55)**.** Supertankers off-load oil at a docking facility 4 mi offshore. The nearest refinery is 9 mi east of the shore opint nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $\$300,000$ per mile if constructed underwater and $\$200,000$ per mile if overland.



(a) Locate Point B to minimize the cost of the construction.

(b) The cost of underwater construction is expected to increase, whereas the cost of overland construction is expected to stay constant. At what cost does it become optimal to construct the pipeline directly to the refinery?

Problem 2 (Thomas §4.1 # 59)**.** The function

$$V(x) = x(10 - 2x)(16 - 2x) \quad \text{for} \quad 0 < x < 5$$

models the volume of a box.

(a) Find the extreme values of V.

(b) Interpret any values found in part **(a)** in terms of volume of the box.

Problem 3 (Thomas §4.1 # 66)**.** If an even function $f(x)$ has a local maximum at $x = c$, can anything be said about the value of f at $x = -c$? Justify your answer.

Problem 4 (Thomas §4.1 # 67)**.** If an odd function $g(x)$ has a local maximum at $x = c$, can anything be said about the value of g at $x = -c$? Justify your answer.

Problem 5 (Thomas §4.1 # 69)**.** Consider a generic cubic function

$$f(x) = ax^3 + bx^2 + cx + d.$$

(a) Show that f can have 0, 1, or 2 critical points. Give examples and graphs to support your argument.

(b) How many local extreme values can f have?

Problem 6. The *unit circle* is
$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}.$$
Find the equation of the line which is tangent to the unit circle at the point $(\cos\theta, \sin\theta)$, where $\theta = \dfrac{2\pi}{3}$.

Problem 7 (Thomas §3.6 # 30)**.** Consider the equation
$$x + \sin y = xy.$$
Use implicit differentiation to find dy/dx.

Problem 8 (Re: Thomas §3.6 # 30)**.** Consider the equation
$$y + \sin x = xy.$$

(a) Solve for y so that y is a function of x. Let $f(x) = y$.

(b) Graph your function on a graphing calculator, and sketch the graph.

(c) What is the domain of f?

(d) Where does the equation $y + \sin x = xy$ implicitly define y as a function of x?

(e) Where does the equation $x + \sin y = xy$ implicitly define x as a function of y?

Problem 9. Compute
$$\lim_{h\to 0} \frac{\sin(a + h) - \sin a}{h},$$
where $a = \pi/3$.

Problem 10 (*Challenge*)**.** Let $f(x) = x^4 + 48x + 1$ and $g(x) = 4x^3 + 8x^2 - 7$. Find all $x \in \mathbb{R}$ such that the graphs of f and g have parallel tangent lines at x.